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Defining and Testing the Influence of Servo System Response on Machine Tool Compliance

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Abstract
Compliance can be defined as the measurement of displacement per unit of force applied e.g. nano-meters per Newton (m/N). Compliance is the reciprocal of stiffness. High stiffness means low compliance and visa versa. It is an important factor in machine tool characteristics because it reflects the ability of the machine axis to maintain a desired position as it encounters a force or torque. Static compliance is a measurement made with a constant force applied e.g. the average depth of cut. Dynamic compliance is a measurement made as a function of frequency, e.g. a fast too servo (FTS) that applies a varying cutting force or load, interrupted cuts and external disturbances such as ground vibrations or air conditioning induced forces on the machine. Compliance can be defined for both a linear and rotary axis of a machine tool. However, to properly define compliance for a rotary axis, the axis must allow a commanded angular position. Note that this excludes velocity only axes. In this paper, several factors are discussed that affect compliance but emphasis is placed on how the machine servo system plays a key role in compliance at low to mid frequency regions. The paper discusses several techniques for measuring compliance and provides examples of results from these measurements.

Compliance Factors

Compliance is a function of several machine characteristics among these are the machine mechanics, geometry, control system, control system type and slide mass. Machine mechanics can be considered to be the deformation of the mechanical hardware to an applied force. Any hardware deformation that can not be measured by the control system will lead to an increase in machine compliance. Consider, for example, a cutting tool with a 3/8-inch cutting shank. Even with an infinitely stiff machine (low compliance); several pounds of cutting force applied to the tool will cause the tool shank to distort several micro-inches. This distortion is not measured by the control system so the tool will not be at the desired or commanded position.

The geometry of the machine tool actuators and feedback sensors affect the machine compliance. Consider, for example, the location of the cutting tool with respect to the machine sensors and actuators. If the tool is located off centerline of the machine motion sensors (Abbe error) and actuators, then a force displacing the tool may not be properly measured by the feedback sensor. The actuator will therefore not move the proper amount to compensate for the displacement. The tool force has caused a yaw of the axis and the actuator can not correct for the yaw, the tool is displaced from the desired position and compliance increases.

If the mechanical and geometric factors of compliance are understood, then the machine control system and slide mass (or rotary axis inertia) have the greatest affect on machine compliance. Machine compliance can be broken down into three frequency ranges; low, medium and high. In the low frequency range, the control system dominates the compliance response. At the mid frequency range, the control system no longer contributes to improvements in compliance and at the high frequency range the mass of the slide dominates the compliance response. The effect of mass on the high frequency range can be found by considering a constant sinusoidal force of excitation driving the slide. Let the measured displacement (x) take the form of $A\sin(\omega t)$ then velocity (x dot) must be $A\omega\cos(\omega t)$ and acceleration (x double dot) must be $-A\omega^2\sin(\omega t)$. Since force is equal to mass times acceleration ($F = ma$) and ω is equal to $2\pi f$, to keep a constant force implies a reduction in the magnitude of A by $1/f^2$ as the frequency increases. By rearranging this equation and solving for frequency, it can be shown that increasing the mass of the slide will lower the frequency, assuming the same slide displacement and force of excitation. In other words, a heavier slide will have a lower compliance at the high frequency range.

The machine control system servo response plays the largest role in machine compliance at low to mid frequencies. In particular, it is the control system loop gain that allows the machine to reject a disturbing force. Since loop gain is a function of frequency, so is machine compliance. In the region of the effectiveness of the control system, compliance and disturbance rejection are strongly related. The transition between the low frequency region and the medium frequency region occurs at the crossover frequency of the control system loop gain. The transition between the mid-frequency region and the high frequency region is a function of the axis mass or the rotary axis inertia. Note that just like the control system loop gain; compliance is a vector quantity it has both a magnitude and phase. A bode plot can provide an accurate reflection of compliance as a function of frequency.

Static compliance of a machine tool is a function of the type of control system. A type zero control system implies there is no integration in the position loop and hence static compliance is a measurable quantity. A type one control system implies that there is an integrator in the position loop and hence, theoretically, there should be no measurable compliance. (Note that this is valid as long as the control system can supply the restoring force or torque and the limit of control system operating range is not exceeded.). Most diamond-turning machines use this type of control system.

Setup and Compliance Measurements:
Static:
Measurements of static compliance of a machine tool are made at zero Hertz. Consider the case of a constant force applied to a machine axis, a measurement of how far the axis moves from the desired command position divided by the applied force represents the static compliance of the machine. This measurement is fairly easy to make. A fish scale or force gauge is used to push or pull on the slide with a constant force while the slide displacement is measured. This measurement can be made with an external position sensor or by observing the slide displacement following error. (Point A Figure 1.) [Note it is important to observe the type of control system and to account for the mechanical and geometric factors that influence this measurement.] A complete measurement of static compliance should be done at several different force values to assure that the results are linear or nearly a linear function of force. A practical example of force induced displacement error can be the result of a dull tool or of taking greater and greater depths of cut. A large force or large depth of cut and high static machine compliance implies that the actual tool position is not equal to the commanded tool position.

Dynamic:
Dynamic compliance reflects the fact that compliance changes as a function of frequency. It is a more difficult measurement to make than a static measurement[1]. In this paper, the servo system is used to make dynamic compliance measurements. Two techniques are discussed to make the measurements. Both techniques take advantage of the already built-in machine slide actuator to cause a force disturbance, and by observing the following error, find the machine axis displacement. These techniques are fairly easy to setup on many machines and do not require setting up any additional mechanical hardware on the machine to create a frequency dependant forcing function for the machine axis. In addition, setting up a displacement sensor to measure the axis displacement is also not required. Although the measurement results may not be the final value of compliance at the tool (see section - Compliance Factors), the measurement can be used to provide a relative value of the machine tool compliance.

Figure 1 will be used to describe the servo compliance measurement setup. It shows a typical servo axis configuration for a machine tool. Although a rotary axis is shown in the diagram using a torque amplifier driving a brush motor, it is equally relevant to a linear axis using a brush or brushless motor. The difference occurs in the type of position feedback device or in the type of actuator or actuator driver. For example, data presented later in this paper is derived from a linear axis that uses a scale for position feedback and a controller commutated two phase input brushless force amplifier to drive a brushless motor. Although the compliance setup for the

brushless motor is a bit more difficult to setup, the compliance measurement tests work equally well for any type of actuator and slide type. However, for this technique to work, the most important factor is that the amplifier be a torque of force amplifier and not a velocity amplifier. Measuring compliance is then found by exciting a torque/force input at point F and measuring the change in displacement at point B or the following error at point A.
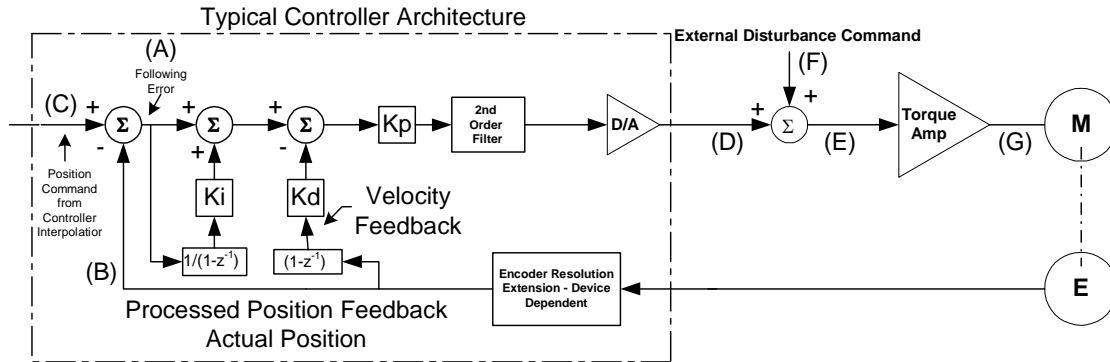

Figure 1. Typical machine tool control architecture with measurement functions for dynamic compliance

Important Amplifier Characteristics for Dynamic Compliance
Since compliance measurement techniques described in this paper rely on the built in slide actuator to provide force disturbance, it is important to understand the limitation of the actuator and most importantly the actuator driver (Force/Torque amplifier. A voltage to current converter). In most cases, the bandwidth of the amplifier is probably an order of magnitude higher than the crossover frequency of the control system. This means that it will not be the limiting factor on servo performance and should not affect the compliance measurements. However, since the measurement techniques depend on the ability to apply a constant disturbance force to the system throughout the frequency range of the test, it is important to understand these limitations. A constant torque/force is applied to the system by inserting a constant amplitude signal at point F. It is assumed that the amplifier frequency response is constant throughout the frequency range of the tests. However, if there is any question about the amplifier response, it can be checked the following way. Open the servo loop at point D. Clamp the axis so it cannot move. Place a current probe at point G and measure the transfer function of G versus F. This is the amplifiers dynamic response and represent how well the constant amplitude signal applied at point F is reflected as a constant torque/force.

Measurement Technique #1:
The process of measuring dynamic compliance can be done in several different ways depending on the controller and the availability of instrumentation. Both techniques rely on finding a calibrated forcing function, however, the displacement data or following error is acquired by two different methods for technique #1.

To find the calibrated forcing function, a summing junction is inserted between the D/A output of the controller (point D) and the torque amplifier input (point E) (See Figure 1.). Since the amplifier is a torque amplifier, the voltage at point D or E (in the absence of a signal at point F) is proportional to the motor current and hence the torque applied to the axis. For the purpose of this discussion, it is assumed that a type one control system is in use, i.e. there is an integrator active in the position loop. This tends to force the following error [Commanded – Actual Position, Point (A) in Figure 1] to zero. Therefore, in the absence of mechanical and geometrical compliance, the static compliance of this axis is zero (Infinitely stiff). Hence, a torque applied to the axis will cause the control system to apply a restoring torque (voltage change) at point D necessary to maintain axis position. Knowing the applied torque, and measuring this voltage change, it is possible to

obtain volts per torque value or calibration constant. (Note that it may be possible to calculate this value from the amplifier and motor data sheets but the method presented here assumes this data is unavailable or unknown by the test operator.)

Once the above value is found, it is now possible to generate a sinusoidal signal with a peak value equal to the magnitude of this constant. This signal can now be inserted at point F in Figure 1 to provide a known disturbance force and can be swept in frequency throughout the desired frequency range of the compliance test. The response of the peak following error observed at point A versus the applied peak torque is the dynamic compliance at the disturbance frequency.

Technique #1: Measurement data acquisition method A:
To acquire the displacement data for these techniques, it is necessary for the controller to allow access to the following error. Additionally, some modern day machine tool controllers allow the following error data to be acquired with a data-gathering program over a period of time. This feature can be used to acquire the data while a calibrated forcing function (voltage) is applied at point F in Figure 1. It is important to be sure the data-gathering program will sample at a sufficient rate to avoid aliasing and be able to indicate the peak value of the following error for the peak force applied at the forcing frequency.
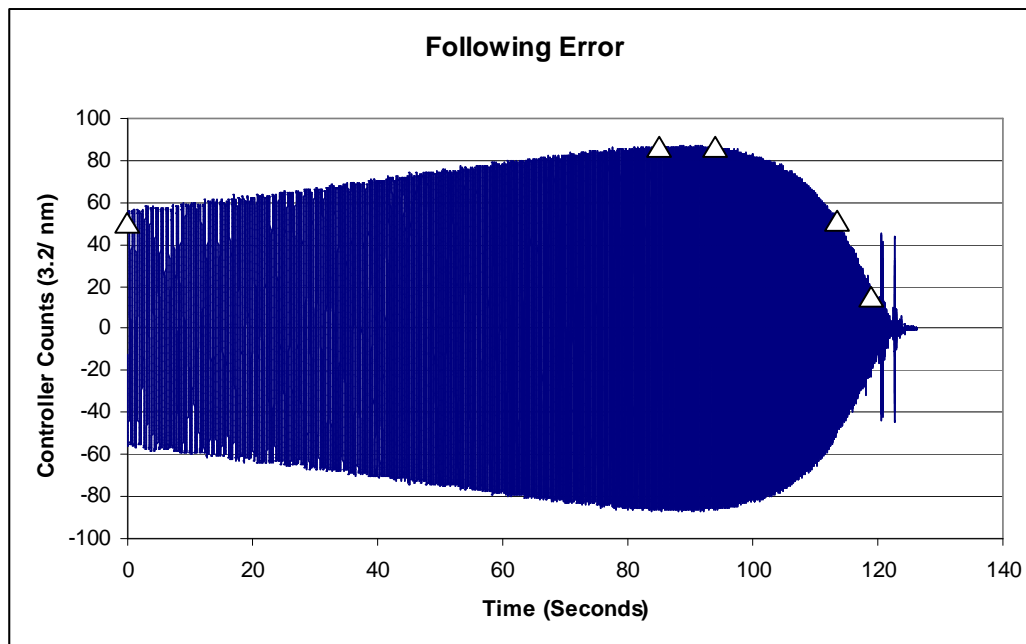
Figure 2: Following error with a constant magnitude of force disturbance applied to the axis as the frequency of the disturbance increases logarithmically from 2 Hz to 5 kHz.

Frequency (Hz)	Controller counts	Nano-meters	Est. Time @ freq. (sec)
2	48	154	0
8	82	262	86
10	83	266	95
30	55	176	113
100	14.2	45	117
163	42	134	n/a

Figure 3: Point by point measurement of the following error at the specified frequencies

The data acquisition method A uses a dynamic signal analyzer (DSA). The DSA is set to sweep the desired frequency range of the compliance test. Additionally, the analyzer output is set to

produce a constant amplitude sinusoidal signal (constant force disturbance). The signal is inserted at point F in Figure 1. The data-gathering program of the controller is setup to observe the following error. The data gathering time should be chosen as close as possible to the time it takes the analyzer to complete the frequency sweep. Start the analyzer sweep and the data-gathering program at the same time. The results of the data-gather buffer scaled in units of displacement divided by the force applied, is the dynamic compliance. The problem with this approach is the trade off between the data-gathering sample rate and the length of the buffer. For an accurate result, the peak of the following error and the frequency must be clearly determined from the gathered data. This may be difficult to do properly, the sweep range may take a long period of time to complete requiring a low sample rate but a low sample rate will result in data aliasing at higher frequencies of the sweep range. A high sample rate may overflow the buffer before the sweep completes. The data from such a measurement is shown in Figure 2. In this case, the Y scale is in units of controller counts, it must be scaled by the conversion factor of 3.2 nano-meters per count to calculate displacement. The excitation source was 220 mV peak. This corresponds to a peak force of the disturbance of one pound. The sweep frequency started at 2 Hz and ended at 5 kHz. The time to complete the sweep was approximately 126 seconds. The data sample rate was 2.25 kHz. The sweep rate was logarithmic.

Technique #1: Measurement data acquisition method B:
The second approach to acquiring the data is to use a fixed frequency disturbance at point F (see Figure 1) e.g., by using a signal generator. The data-gathering sample rate can be chosen to properly acquire the following error signal at the disturbance frequency without alias and in a short time period per point. The compliance can be calculated as before. Since a bode plot is a desirable result of a dynamic compliance test, it is possible to repeat this test at several frequencies and build a magnitude bode plot. However, since the generator and the data gathering program are not synchronized, phase information is not available. Figure 3 is a chart of data obtained in this way. The point by point data is also plotted on in Figure 2 and shows very close agreement with both measurement methods of technique one.

Technique #2:
Technique two requires a similar setup to technique one method measurement A but there is no need to use the data gathering routine. This technique relies on the ability to write the following error (point A) to a DAC (D/A). If the controller does not allow direct access to point A of Figure 1, it may be possible to mathematically calculate the following error. It is the commanded position (point C) minus actual position (point B). Writing this result to a DAC with a bias term that allows the DAC output to be set to zero volts allows the DAC output to be set to zero volts at any test position. This may be done by taking advantage of controller supported routines[2]. The DSA can be set to take a transfer function of the following error (D/A output) divided by the signal (applied disturbance force) at point F. The advantage of this method is that both the magnitude and phase data are available. The DSA provides a fast and convenient way of measuring dynamic compliance and can provide a bode plot as output. Figure 4 shows the results of such a dynamic compliance test along with the open and closed loop servo response. The peak of force disturbance during this test was one pound (220mV). The scale in dB can be converted to displacement knowing the conversion factor of DAC volts per controller counts and counts to displacement. So for a 16 bit DAC with full scale output of +/-10 volts, -57.15 dB = 1 count or 3.2 nm. [-57.1 dB = 20 log 10 (20/(2^16)/0.22)]. The peak of the compliance curve is -18.4 dB or the following error is 87 counts or 278 nano-meters.

Conclusion
Static and dynamic compliance are important factors in evaluating a machine tool. Static compliance is a measurement of how well the tool will follow a desired path while the tool is under a constant load. Dynamic compliance is a measurement of how well the machine can reject varying loads or noise forces. In this paper factors are discussed that influence compliance most notably the servo system response. Data is presented from several methods of using the servo system of the machine tool to find dynamic compliance.
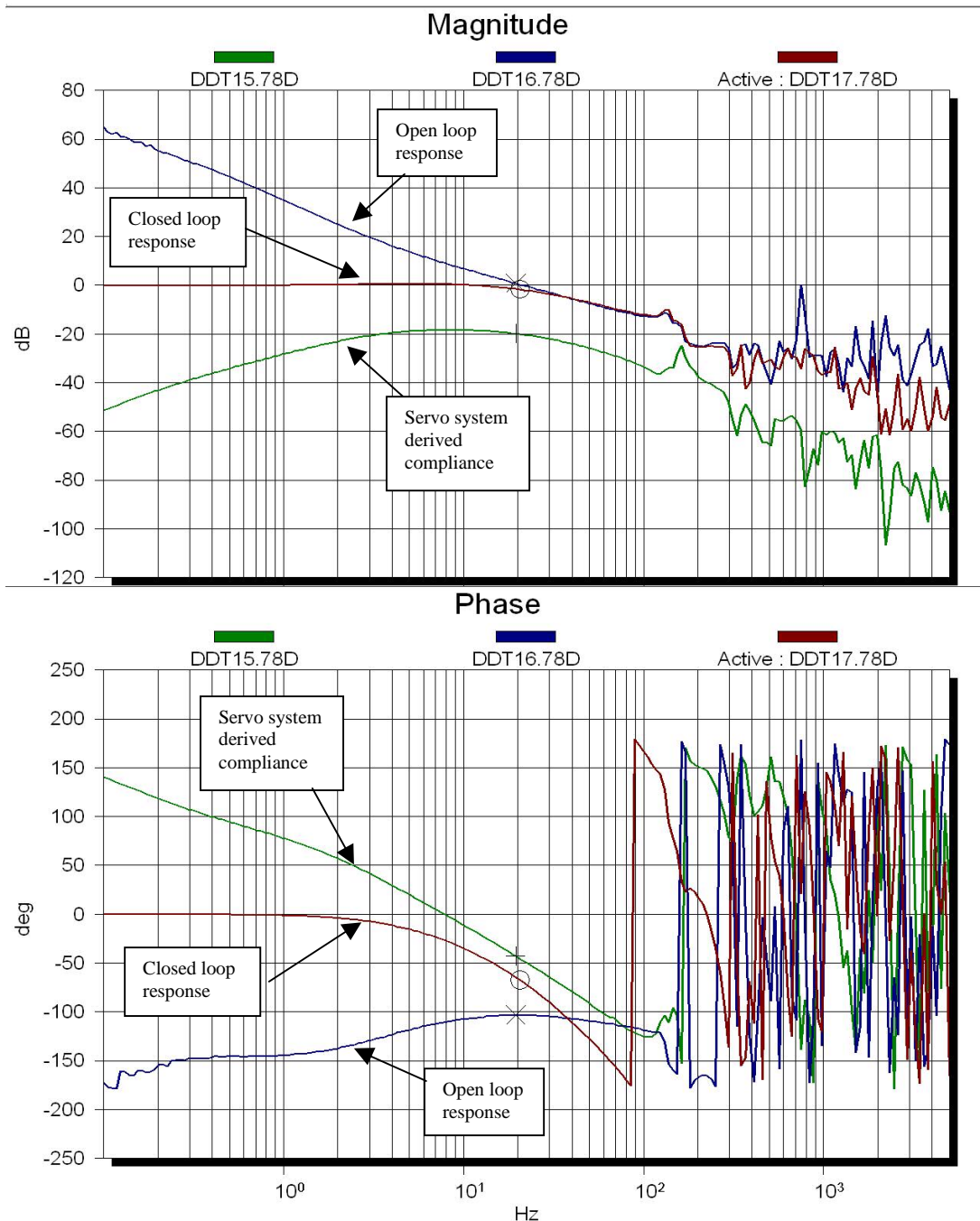
Figure 4: Transfer functions of servo response and compliance response

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